Introduction to Statistics and Data Science using *eStat*

Chapter 8 Testing Hypothesis for Two Populations

8.3 Testing hypothesis for two population proportions

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8.3 Testing hypothesis for two population proportions

- Examples of comparing the two population proportions.

- Is there a gender gap in the approval rating for a particular candidate in this year's presidential election?

- A factory has two machines that make products. Do the two machines have different defect rates?

8.3 Testing hypothesis for two population proportions

- Hypothesis for two population proportions:

1) $H_0 : p_1 = p_2$
   $H_1 : p_1 > p_2$

2) $H_0 : p_1 = p_2$
   $H_1 : p_1 < p_2$

3) $H_0 : p_1 = p_2$
   $H_1 : p_1 \neq p_2$

- Estimator of difference in population means p_1 - p_2

⇨ difference in sample means $\hat{p}_1$ - $\hat{p}_2$

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- Sampling distribution of $\hat{p}_1$ - $\hat{p}_2$, if samples are large

$$\hat{p}_1 - \hat{p}_2 \approx N\left(p_1 - p_2\,,\ \frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}\right)$$

- Test Statistic if $H_0 : p_1 = p_2$ is true

$$\frac{(\hat{p}_1 - \hat{p}_2)}{\sqrt{\frac{\bar{p}\,(1-\bar{p})}{n_1}+\frac{\bar{p}(1-\bar{p})}{n_2}}} \approx N(0,1)$$

where $\bar{p} = \dfrac{n_1\,\hat{p}_1 + n_2\,\hat{p}_2}{n_1 + n_2}$ is weighted average of sample proportions

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Table 8.3.1 Testing hypothesis for two population proportions
- two independent large samples -

Type of Hypothesis	Decision Rule
1) $H_0 : p_1 = p_2$ $H_1 : p_1 > p_2$	If $\dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\dfrac{\bar{p}(1-\bar{p})}{n_1} + \dfrac{\bar{p}(1-\bar{p})}{n_2}}} > z_\alpha$, then reject H_0, else accept H_0
2) $H_0 : p_1 = p_2$ $H_1 : p_1 < p_2$	If $\dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\dfrac{\bar{p}(1-\bar{p})}{n_1} + \dfrac{\bar{p}(1-\bar{p})}{n_2}}} < -z_\alpha$, then reject H_0, else accept H_0
3) $H_0 : p_1 = p_2$ $H_1 : p_1 \neq p_2$	If $\left\lvert \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\dfrac{\bar{p}(1-\bar{p})}{n_1} + \dfrac{\bar{p}(1-\bar{p})}{n_2}}} \right\rvert > z_{\alpha/2}$, then reject H_0, else accept H_0

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[Example 8.3.1] In this year's presidential election, 54 out of 225 samples from male population supported the candidate A and 52 out of 175 samples from female population supported the candidate A where samples are independent.

- Test whether there is a difference in male and female approval ratings at a 5% significant level.
- Check the results using 『eStatU』

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<Answer of Ex 8.3.1>

- The hypothesis of this proble is H_0 : $p_1 = p_2$, H_1 : $p_1 \neq p_2$ and its decision rule is as follows.

  If $\left| \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\dfrac{\overline{p}(1-\overline{p})}{n_1} + \dfrac{\overline{p}(1-\overline{p})}{n_2}}} \right| > z_{\alpha/2}$, then reject H_0, else accept H_0

- Since $\hat{p}_1$= 54/225 = 0.240, $\hat{p}_2$= 52/175 = 0.297, $\overline{p}$ and the test statistic can be calculated as follows.

$$\overline{p} = (54+52) / (225 + 175) = 106/400 = 0.265$$

$$\left| \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\overline{p}(1-\overline{p})}{n_1} + \frac{\overline{p}(1-\overline{p})}{n_2}}} \right| = \left| \frac{0.240 - 0.297}{\sqrt{\frac{0.265(1-0.265)}{225} + \frac{0.265(1-0.265)}{175}}} \right| = 1.28$$

$$z_{\alpha/2} = z_{0.05/2} = z_{0.025} = 1.96$$

Therefore the hypothesis H_0 cannot be rejected and we conclude that there is not enough evidence to say that the approval ratings of certain male and female candidates are different.

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<Answer of Ex 8.3.1>

Testing Hypothesis p_1, p_2

[Hypothesis] $H_0 : p_1 - p_2 = D$ 0

◉ $H_1 : p_1 - p_2 \neq D$ ○ $H_1 : p_1 - p_2 > D$ ○ $H_1 : p_1 - p_2 < D$

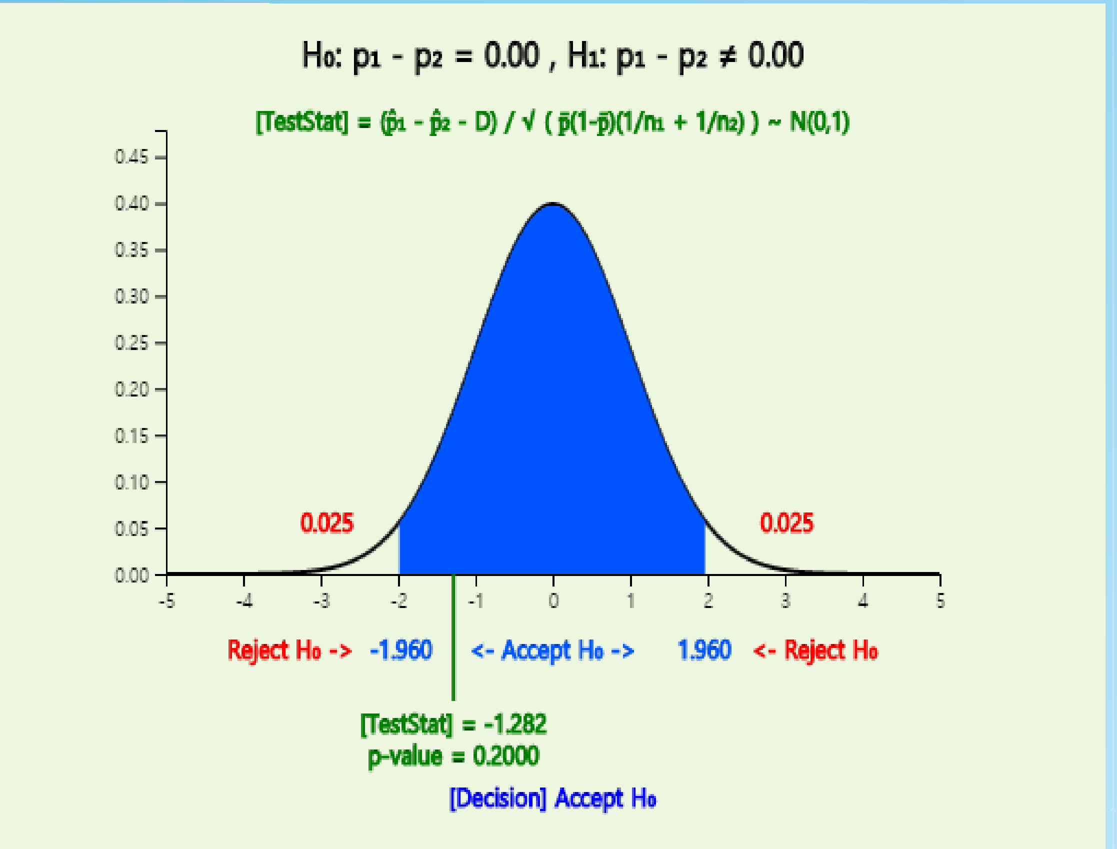
[Test Type] *Z test*

Significance Level $\alpha =$ ◉ 5% ○ 1%

[Sample Data]

Sample Size	n_1 =	225	n_2 =	175
Sample Proportion	$\hat{p}_1$ =	0.240	$\hat{p}_2$ =	0.297

Execute



Thank you