Introduction to Statistics and Data Science using *eStat*

Chapter 7 Testing Hypothesis for Single Population

7.4 Testing Hypothesis with α and β simultaneously

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7.4 Testing Hypothesis with α and β simultaneously

- Testing hypothesis so far is a conservative decision-making method.
  ⇨ critical value that reduces the probability of type one error α
  (error that rejects the null hypothesis even though it is true).
  ⇨ keep H_0 unless there is sufficient evidence of H_1 which is risky.
  ⇨ probability of type two error (β) was not considered.

- Sometimes it is unclear which should be H_0 and which one should be H_1.
  ⇨ both α and β are important and should be considered simultaneously.
  ⇨ If the analyst can determine the sample size,
  testing hypothesis with α and β can be performed.

7.4 Testing Hypothesis with α and β simultaneously

7.4.1 β and the power of test

[Example 7.4.1] In [Example 7.1.1], calculate the probability of the type 2 error β if the significance level is 5%. Check this result using 『eStatU』.

〈Answer〉

- Hypothesis is H_0 : μ = 1500, H_1 : μ = 1600,
- Population standard deviation is assumed σ = 200, and n =30.
- Decision rule is as follows if α is 5%.

'If $\overline{X}$ 〉1500 + (1.645) $\sqrt{\frac{200^2}{30}}$ = 1560.06, reject H_0'

- Type 2 error (probability of H_0 is true when H_1 is true) is calculated as:

β = P($\overline{X}$ 〈 1560.06 | H_1 is true)

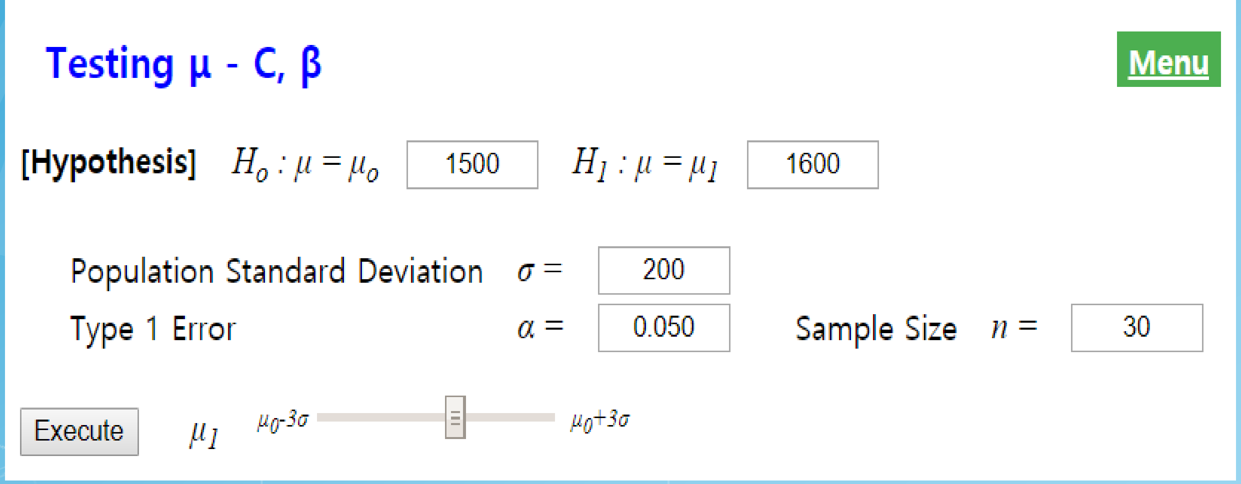
= P(($\overline{X}$ −1600) / $\sqrt{\frac{200^2}{30}}$ 〈 (1560.06−1600) / $\sqrt{\frac{200^2}{30}}$)

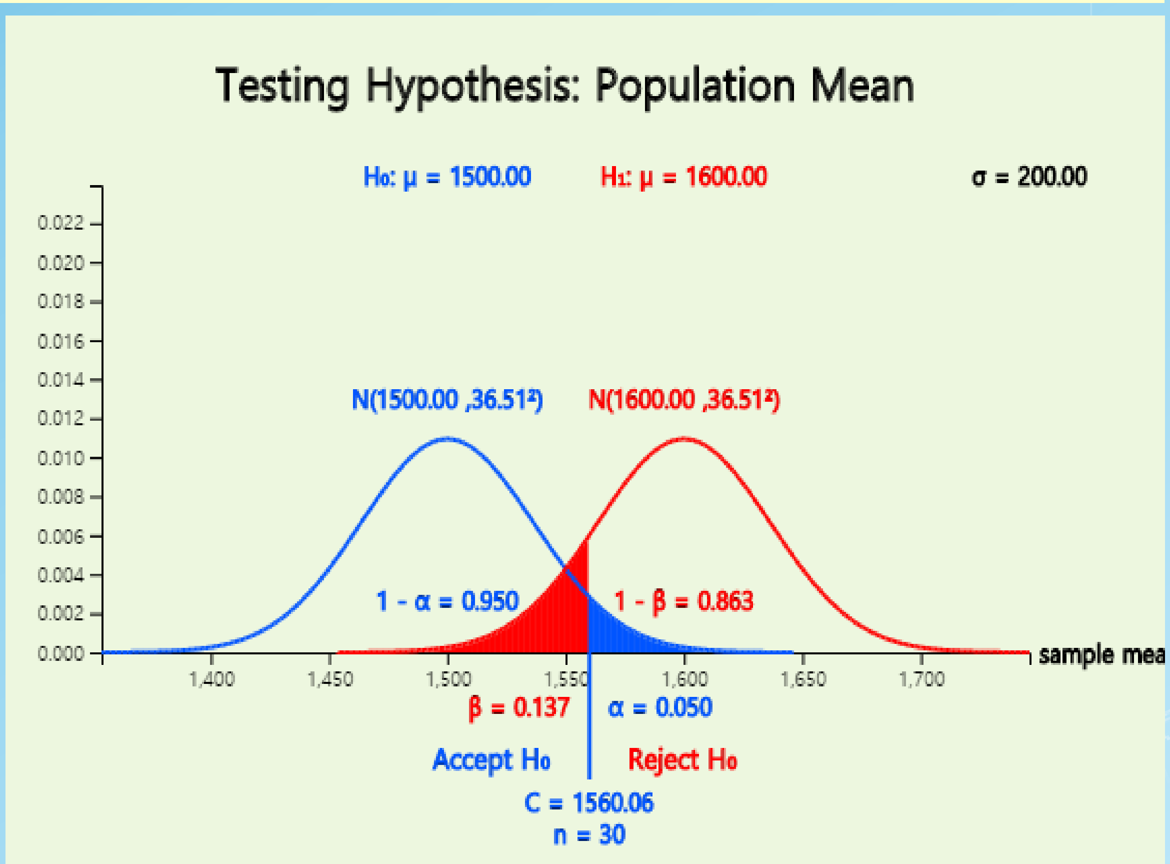
= P(Z 〈 −1.09) = 0.137

7.4 Testing Hypothesis with α and β simultaneously

⟨Answer of Ex 7.4.1⟩ Select 'Testing $\mu - C,\ \beta$' at『eStatU』menu.

- Enter $\mu_0 = 1500$, $\mu_1 = 1600$, σ = 200, $\alpha = 0.05$, n =30 and click [Execute] button.
- Result of testing hypothesis, critical value C and β, will be shown.





7.4 Testing Hypothesis with α and β simultaneously

[Example 7.4.2] In [Example 7.1.1], if the null hypothesis is not changed, but the alternative hypothesis is changed as follows:

H_0: μ = 1500, H_1 : μ = 1580

1) Calculate the probability of the type 2 error if the significance level is 5%.

2) Check this result using 『eStatU』.

〈Answer〉

1) Although the alternative hypothesis has been changed to H_1 : μ = 1580, decision rule will not be changed because H_1 is the ype as H_1 : μ 〉1580.

'If $\overline{X}$ 〈 1500 + (1.645) $\sqrt{\frac{200^2}{30}}$ = 1560.06, reject H_0 '

- Hence, the probability of type 2 error is as follows:

β = P($\overline{X}$ 〈 1560.06 | H_1 is true)

= P(($\overline{X}$ −1580) / $\sqrt{\frac{200^2}{30}}$ 〈 (1560.06−1580)/ $\sqrt{\frac{200^2}{30}}$)

= P(Z 〈 −0.546) = 0.293

7.4 Testing Hypothesis with α and β simultaneously

7.4.1 β and the power of test

- Discriminating ability of two hypothesis is compared by using the power of a test.

  Power = 1 − (Probability of the type 2 error) = 1 − β

  Large power increases discriminating ability of the test.

- Power of a test can be obtained for any μ_1 of $H_1 : \mu = \mu_1$. It means that the power is a function over the value of and it is called a power function.

- A function of the probability that the null hypothesis is correct when the null hypothesis is true is called an operating characteristic function.

  Operating characteristic function = 1 − α

7.4 Testing Hypothesis with α and β simultaneously

[Example 7.4.3] In [Example 7.1.1], calculate the power of the following alternative hypothesis. Use α = 0.05. By using this, approximate the power function.

1) H_1 : μ = 1500	2) H_1 : μ = 1510	3) H_1 : μ = 1520
4) H_1 : μ = 1530	5) H_1 : μ = 1540	6) H_1 : μ = 1550
7) H_1 : μ = 1560	8) H_1 : μ = 1570	9) H_1 : μ = 1580
10) H_1 : μ = 1590	11) H_1 : μ = 1600	12) H_1 : μ = 1610

〈Answer〉

1) Although the alternative hypothesis are different, decision rule will not be changed because H_1 is the ype as H_1 : μ 〉1580.

‘If $\overline{X}$ 〈 $1500 + (1.645)\sqrt{\frac{200^2}{30}} = 1560.06$, reject H_0 ’

- Hence, if we calculate the probability of the type 2 error as [Example 7.4.2], the power of each test is as follows:

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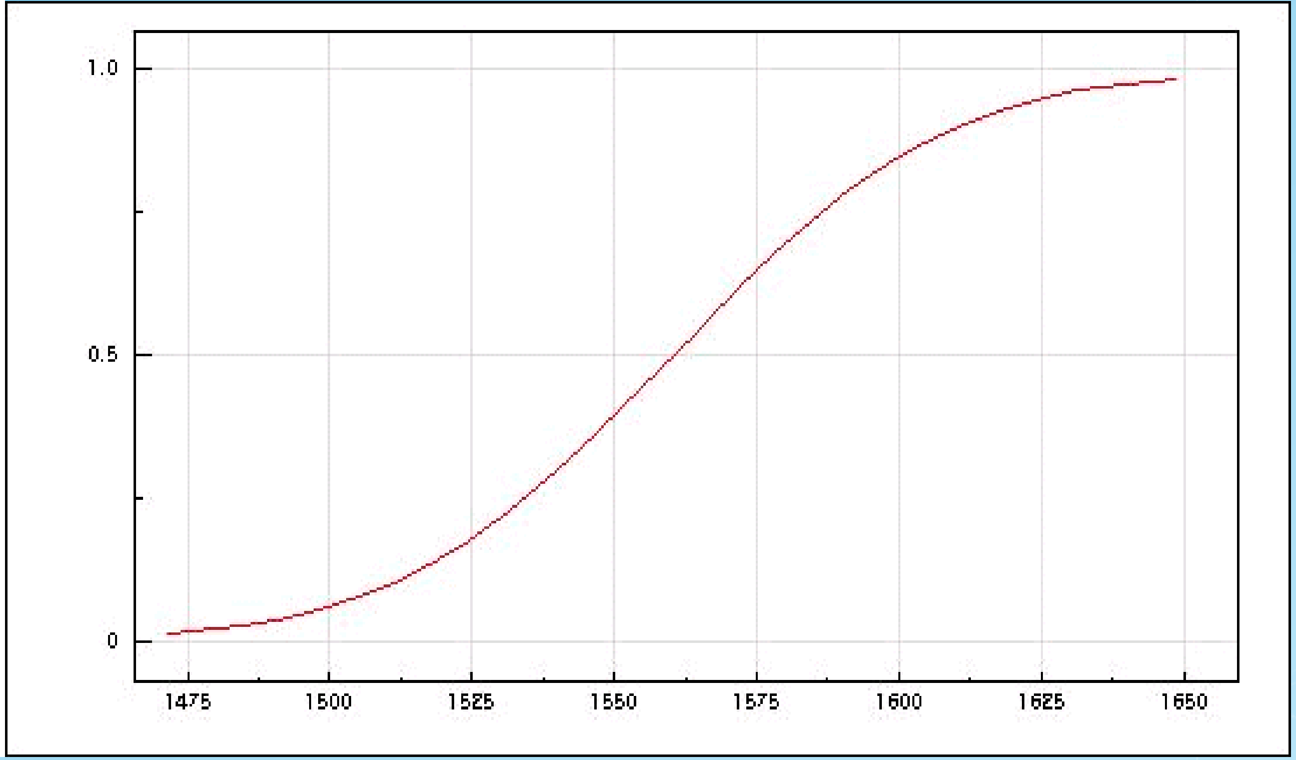
⟨Answer of Ex 7.4.3⟩

Alternative Hypothesis	β	Power = 1 - β
1) H_1 : μ = 1500	0.95	0.05
2) H_1 : μ = 1510	0.91	0.09
3) H_1 : μ = 1520	0.86	0.14
4) H_1 : μ = 1530	0.79	0.21
5) H_1 : μ = 1540	0.71	0.29
6) H_1 : μ = 1550	0.61	0.39
7) H_1 : μ = 1560	0.50	0.50
8) H_1 : μ = 1570	0.39	0.61
9) H_1 : μ = 1580	0.29	0.71
10) H_1 : μ = 1590	0.21	0.79
11) H_1 : μ = 1600	0.14	0.86
12) H_1 : μ = 1610	0.09	0.91

- **Power function**



7.4 Testing Hypothesis with α and β simultaneously

7.4.2 Testing Hypothesis with α and β

[Example 7.4.4] Consider the testing hypothesis on the bulb life such as H_0: μ = 1500, H_1 : μ = 1570. Find sample size n and decision rule which satisfies α = 5% and β = 10%. Assume σ = 200 hours.

〈Answer〉

- Let n be the sample size and C be the critical value of a decision rule.
- Probability of the type 1 error α and type 2 error β are defined as follows:

α = P($\overline{X} > C$ | H_0 is true)
β = P($\overline{X} < C$ | H_1 is true)

7.4 Testing Hypothesis with α and β simultaneously

〈Answer of Example 7.4.4〉

- If H_0 is true, $\bar{X} \sim N(1500, \frac{200^2}{n})$ and if H_1 is true, $\bar{X} \sim N(1570, \frac{200^2}{n})$. If α = 0.05 and β = 0.10, then $z_{0.05}$ = 1.645 and $z_{0.10}$ = −1.280. Hence n and C should satisfy both of the following equations.

$$C = 1500 + 1.645 \times \frac{200}{\sqrt{n}}$$
$$C = 1570 - 1.280 \times \frac{200}{\sqrt{n}}$$

- By solving two system of equations, solution is n = 69.8, C = 1539.4. i.e., the sample size is 70 approximately and the decision rule is as follows:

‘If $\bar{X}$ 〉 1539.4, then reject H_0’

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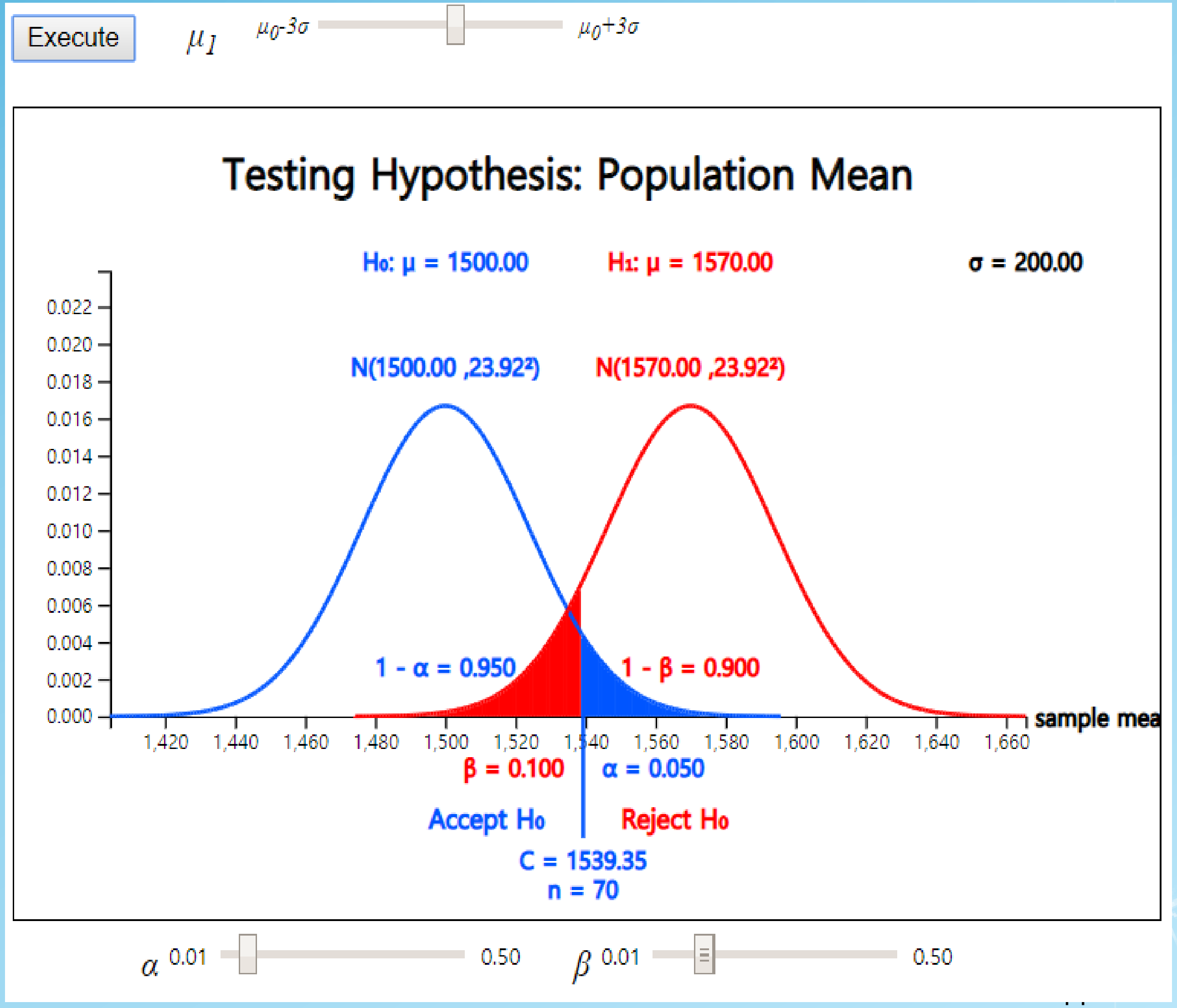
〈Answer of Ex 7.4.4〉

Testing μ - C, n Menu

[Hypothesis] $H_0 : \mu = \mu_0$ 1500 $H_1 : \mu = \mu_1$ 1570

Population Standard Deviation $\sigma =$ 200

Type 1 Error $\alpha =$ 0.050 Type 2 Error $\beta =$ 0.100



Thank you